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MAGNETOPHORETIC POTENTIAL OF A PLANE-ORDERED SYSTEM OF FERROCYLINDERS. I. CIRCULAR CYLINDERS

B. É. Kashevskii

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The magnetophoretic potential of a system of equidistant identical ferrocylinders packed in one plane and exposed to a uniform magnetic field is studied. The influence of the structural step and the direction of magnetization of the structure are investigated.

With the use of a magnetic field combining a high intensity and a strong small-scale nonuniformity one can separate very small weakly magnetic objects, including particles of cell suspensions, from the flow of a gas and a liquid. The method of high-gradient magnetic separation has attracted attention in many spheres of activity, including water and gas purification, purification of clays, chemical technologies, medicine, and biology, since the mid-1970s (see [1–8]).

High-gradient magnetic filters are created in practice by application of a strong uniform magnetic field to a volume in which small ferromagnetic bodies are distributed. The smaller the separated particles and the weaker their magnetic properties, the stronger must be the external field and the smaller the size of the elements of the ferromagnetic packing. Thus, for separation of red blood cells one uses bundles of ferromagnetic wire about 100 μm in diameter [2, 5]. In flow systems, the efficiency of such a filtering structure is low since the probability of capture of particles is low and decreases with increase in the pumping rate. Ordered (regular) filtering structures consisting of a periodic packing of ferrocylinders have been created in a number of investigations [5, 8]. However there is no theoretical analysis of the efficiency of the periodic structure and its optimum configuration, and this analysis is the aim of the present investigation. It is based on the noninductive approximation for description of the field distribution (the mutual influence of the cylinders on the magnetization of each other is excluded) and on the notion of a magnetophoretic potential averaged along the direction of motion of a suspension of separated particles.

We consider a filtering structure representing a set of identical ferromagnetic cylinders of radius a, located in one plane with step S. Let us introduce the Cartesian coordinates X, Y, Z; the axes of the cylinders lie in the plane XY and are directed along Y. The origin of the coordinate system is located on the axis of one cylinder to which a zero number is applied. We assume that the packing occupies the entire plane. The geometry of the packing is totally determined by a set of the radius vectors of the cylinders' axes, $\mathbf{R}_\alpha = \alpha S\mathbf{i}$.

The uniform external magnetic field $\mathbf{H}_0 = H_0\mathbf{e}$ is applied across the packing plane ($\mathbf{e} = \mathbf{k}$) or in the packing plane perpendicularly to the cylinders' axes ($\mathbf{e} = \mathbf{i}$). On condition that the system is magnetized to saturation we can disregard the mutual influence of the cylinders on the magnetization of each other. In this case, the self-field of the filtering structure (distortion of the external field) is determined by the sum of the fields of individual cylinders.

The intensity of the field of a transversely magnetized infinite cylinder (with a number α) on the line A(x, y) beyond the cylinder is given by the relations

$$\mathbf{H}'_\alpha(\mathrm{A}) = 2\pi M_s \mathbf{h}_\alpha\,, \quad \mathbf{h}_\alpha(\mathrm{A}) = -\frac{1}{r_{\alpha\mathrm{A}}^2}\left[\mathbf{e} - \frac{2}{r_{\alpha\mathrm{A}}^2}(\mathbf{e}\mathbf{r}_{\alpha\mathrm{A}})\,\mathbf{r}_{\alpha\mathrm{A}}\right], \quad \mathbf{r}_{\alpha\mathrm{A}} = (x - \alpha s)\,\mathbf{i} + z\mathbf{k}\,, \tag{1}$$

in which the distances x, z, and $r_{\alpha\mathrm{A}}$ are measured in the radii of the cylinder. The total field of the structure will be written in the form

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 6, pp. 70–74, November–December, 2003. Original article submitted March 3, 2003.

$$\mathbf{H}(\mathrm{A}) = \mathbf{H}_0 + \mathbf{H}'(\mathrm{A}), \quad \mathbf{H}'(\mathrm{A}) = 2\pi M_s \mathbf{h}(\mathrm{A}), \quad \mathbf{h}(\mathrm{A}) = \sum_{\alpha} \mathbf{h}_{\alpha}(\mathrm{A}). \tag{2}$$

If the filtering structure is immersed in a suspension of separated particles, a small (as compared to the cylinder diameter) particle possessing a magnetic susceptibility χ, having a volume v, and located in a continuum with a susceptibility χ_0 is acted upon, in a nonuniform magnetic field, by the force

$$\mathbf{F} = \frac{1}{2} \Delta\chi v \nabla H^2, \tag{3}$$

where $\Delta\chi = \chi - \chi_0$. Taking into account the uniformity of the external field, we reduce (3) to the form

$$\mathbf{F} = \frac{1}{2} \Delta\chi v (2\pi M_s)^2 \nabla \left[h^2 + \frac{H_0}{\pi M_s} \mathbf{eh} \right]. \tag{4}$$

Next, following [9], we introduce the magnetophoretic potential of the system according to $\mathbf{F} = -\nabla\Phi$:

$$\Phi = -\frac{1}{2} \Delta\chi v (2\pi M_s)^2 \left[h^2 + \frac{H_0}{\pi M_s} \mathbf{eh} \right]. \tag{5}$$

Employing, as the scale, the value of the magnetophoretic potential of the self-field of an individual cylinder at its forward point

$$\Phi^* = \frac{1}{2} \Delta\chi v (2\pi M_s)^2,$$

we write the dimensionless potential $\varphi = \Phi/\Phi^*$ in the form

$$\varphi = \varphi_1 + \varphi_2, \tag{6}$$

where

$$\varphi_1 = -\mathbf{h}^2, \quad \varphi_2 = -P\mathbf{eh}, \quad P = \frac{H_0}{\pi M_s}. \tag{7}$$

It is noteworthy that the quantity Φ^* employed for making the potential dimensionless can be positive or negative together with the quantity $\Delta\chi$, whose sign depends on the magnetic properties of the carrier liquid. However, for any practical purposes it is convenient to consider $\Delta\chi$ as the effective susceptibility of particles suspended in a nonmagnetic medium and to speak of paramagnetic ($\Delta\chi > 0$) and diamagnetic ($\Delta\chi < 0$) particles. Then the magnetophoretic potential made dimensionless by the method adopted refers to paramagnetic particles, and that taken with an opposite sign refers to diamagnetic particles. In other words, paramagnetic particles move toward the minimum of the potential φ while diamagnetic particles move toward its maximum.

The quantity φ_1 expresses the magnetophoretic potential of the self-field of the filtering structure. In particular, if the cylinders are manufactured from a hard magnetic material with remanence M_s, the quantity φ_1 comprises the total potential of the system with a switched-off external field. The quantity φ_2 represents a result of the interference of the self-field and the external field. We define φ_1 and φ_2 as the intrinsic and interference magnetophoretic potentials. There are significant qualitative differences between them. First, the intrinsic potential is strictly negative and hence has the attraction of paramagnetic particles to the structure and the repulsion of diamagnetic particles as its effect. The interference potential can change its sign at different points above the structure, thus creating prerequisites for sedimentation of diamagnetic particles. Second, the value of the intrinsic potential of the structure (naturally, on attainment of magnetic saturation) is independent of the external-field intensity, whereas the interference potential increases in proportion to H_0. We recall that consideration is given to the state of magnetic saturation of cylinders. Since

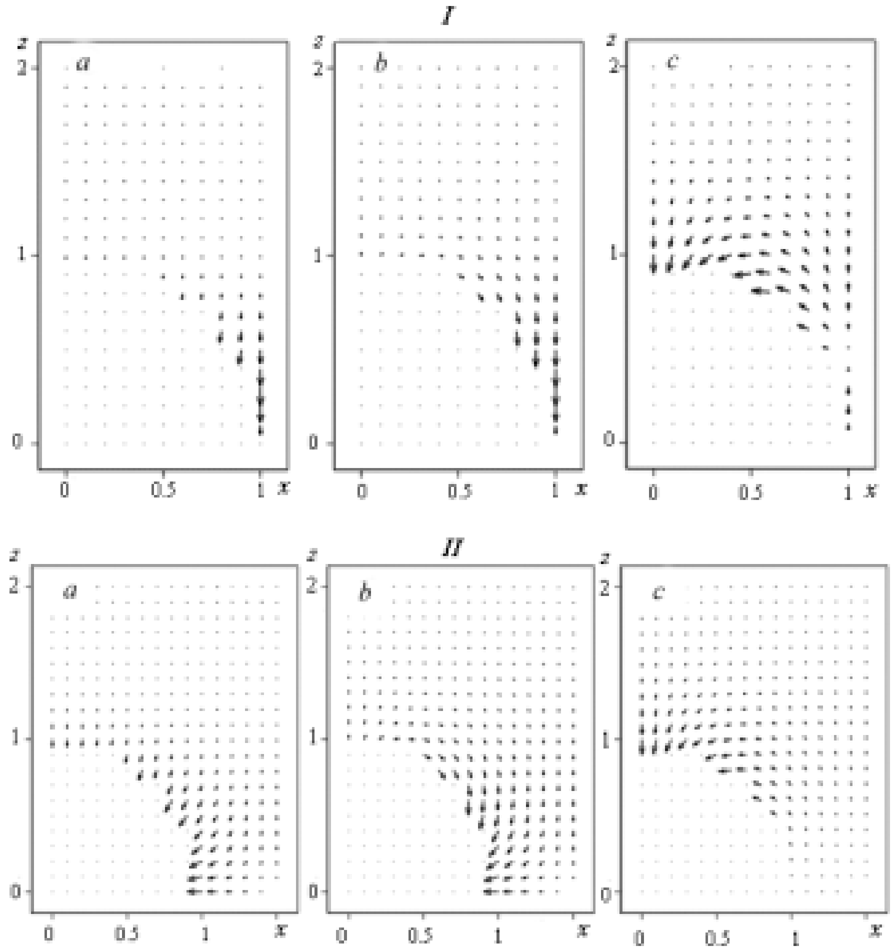


Fig. 1. Field of magnetophoretic force for dense (s = 2) (I) and sparse (s = 4) (II) packings of cylinders; a) force field of the intrinsic potential (P = 0); b and c) force fields of the longitudinally and transversely magnetized system (P = 3).

the saturation is attained in the external field exceeding the degaussing field of the cylinder magnetified to saturation ($H_0 > 2\pi M_s$), relation (6) should be considered for $P \geq 2$. In particular, a field of 10 kOe is required for saturation of cylinders of pure iron (M_s = 1700 G).

Employing the relations obtained, we consider the magnetophoretic properties of a system magnetized in the plane or across the plane of the magnetic structure. Because of the periodic arrangement of the cylinders, the value of the potential is repeated along x at a distance equal to a structural step. Therefore, it is sufficient to consider the region $-s/2 \leq x \leq s/2$.

Let us take the case of longitudinal magnetization (**e** = **i**). Introducing the relative coordinates $\hat{x} = x/s$ and $\hat{z} = z/s$, we write the nonzero components of the self-field (2) in the form

$$h_x^{(\parallel)} = s^{-2} N(\hat{x}, \hat{z}) , \quad N(\hat{x}, \hat{z}) = \sum_{\alpha=-\infty}^{\infty} \frac{(\alpha - \hat{x})^2 - \hat{z}^2}{[(\alpha - \hat{x})^2 + \hat{z}^2]^2} ;$$

$$h_z^{(\parallel)} = s^{-2} T(\hat{x}, \hat{z}) , \quad T(\hat{x}, \hat{z}) = 2\hat{z} \sum_{\alpha=-\infty}^{\infty} \frac{\hat{x} - \alpha}{[(\alpha - \hat{x})^2 + \hat{z}^2]^2} . \tag{8}$$

The infinite sums in these relations can be represented in the form of a combination of elementary transcendental functions $A(\hat{x}) = \cot(\pi\hat{x})$ and $B(\hat{z}) = \coth(\pi\hat{z})$:

$$N = \frac{\pi^2 (A^2 - B^2)(1 + A^2)(1 - B^2)}{(A^2 + B^2)^2}, \quad T = \frac{2\pi^2 AB (1 + A^2)(B^2 - 1)}{(A^2 + B^2)^2}. \tag{9}$$

For the components of the magnetophoretic potential of the longitudinally magnetized structure we have

$$\varphi_1^{(\parallel)} = - s^{-4} (N^2 + T^2) = - s^{-4} I, \quad \varphi_2^{(\parallel)} = - Ps^{-2} N, \quad I = \left[\frac{\pi^2 (1 + A^2)(1 - B^2)}{A^2 + B^2}\right]^2. \tag{10}$$

In the case of transverse magnetization ($\mathbf{e} = \mathbf{k}$) we analogously arrive at the result

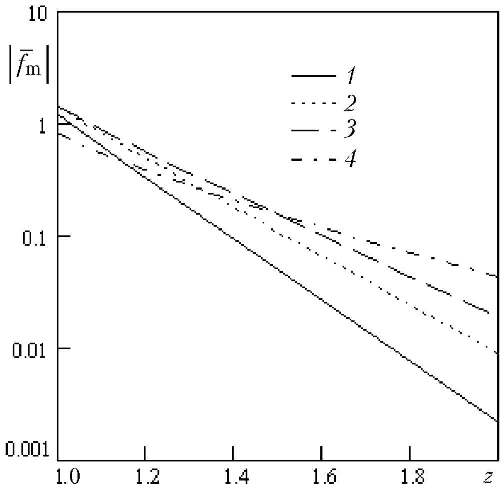
$$h_x^{(\perp)} = s^{-2} T(\hat{x}, \hat{z}), \quad h_z^{(\perp)} = - s^{-2} N(\hat{x}, \hat{z}), \quad \varphi_1^{(\parallel)} = - s^{-4} I(\hat{x}, \hat{z}), \quad \varphi_2^{(\perp)} = Ps^{-2} N(\hat{x}, \hat{z}). \tag{11}$$

Comparing relations (10) and (11), we infer that the intrinsic magnetophoretic potentials of the longitudinally and transversely magnetized structures coincide, whereas the interference potentials change their sign with the direction of magnetization. To obtain a general idea of the magnetophoretic properties of a system we construct the field of the magnetophoretic force $\mathbf{f}_m = -\nabla\varphi$ for different directions of magnetization, structural steps, and magnetizing-field intensity. The field of magnetophoretic force for dense ($s = 2$) and sparse ($s = 3$) packings of cylinders is shown in Fig. 1. As we see, in the remanence field of the cylinders the entire surface of them attracts paramagnetic particles and repels diamagnetic particles. Paramagnetic particles are attracted most intensely in the hollows between the cylinders. In longitudinal magnetization, the attraction of paramagnetic particles is enhanced in hollows and is replaced by repulsion on protrusions (here diamagnetic particles are attracted). We note that the force of diamagnetic attraction on protrusions is much smaller than the force of paramagnetic attraction in hollows. In transverse magnetization of the system, paramagnetic particles, conversely, are attracted to the protrusions whereas diamagnetic particles are pulled into the hollows. Comparing the force of diamagnetic attraction on the protrusions in the case of longitudinal magnetization and that in the hollows in the case of transverse magnetization, we can infer that the latter configuration is more favorable for separation of diamagnetic particles.

In selecting the direction of motion of the suspension of separated particles, one should take into account such a magnetophoretic characteristic of the filtering structure as the magnetophoretic potential averaged along this direction [9]; this potential determines the accumulation of the effect of displacement of particles in their motion in the field of a variable magnetophoretic force. In the system in question, the averaged potential coincides with the running potential $\varphi(x, z)$ in the case of suspension flow along the cylinders, so that we have a monotone movement of particles in the plane of action of the magnetophoretic force (plane x, z) in the process of motion along y. In laminar suspension flow, the velocity field of particles in the plane x, z coincides with the vector fields (Fig. 1) of the magnetophoretic force. We note that in this situation nothing prevents the separated particles from being removed by the liquid flow. Consequently, it can be employed in continuous schemes in which detachment of a part of the particle-enriched flow is organized at the outlet from the filter. Sedimentation of particles in the filter is possible in the case of suspension flow across the cylinders in the x direction. Here, the magnetophoretic force prevents the particles accumulated in the hollows from being removed. Since the direction of the magnetophoretic force changes in the process of movement of particles across the cylinders above the layer surface ($z > 1$), the effect of their separation in the hollows from the external region is determined by the potential average over the line z = const:

$$\overline{\varphi}(z) = \frac{1}{s} \int_{-s/2}^{s/2} \varphi(x, z)\, dx.$$

Let us consider the contribution of the alternating interference component to $\overline{\varphi}$. In the case of longitudinal magnetization it can be represented in the form



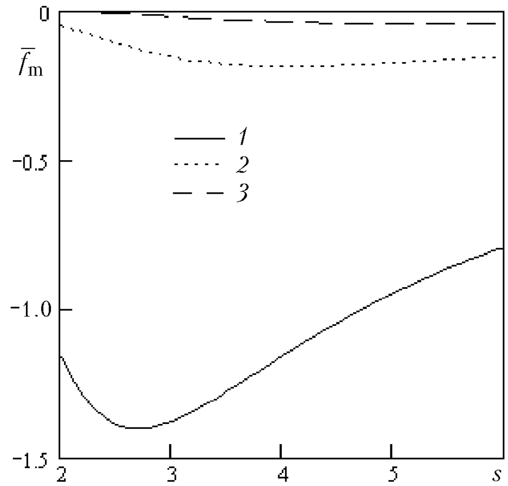


Fig. 2. Absolute value of the average magnetophoretic force $|\bar{f}_m|$ vs. distance z to the plane of the cylinders' axes for different values of the structural step: s = 2 (1), 2.5 (2), 3 (3), and 6 (4).

Fig. 3. Average magnetophoretic force $\bar{f}_m$ vs. structural step at different distances to the plane of the cylinders' axes: z = 1 (1), 1.5 (2), and 2 (3).

$$\bar{\varphi}_2 = -\frac{P}{s}\int_{-s/2}^{s/2}\sum_{\alpha=-\infty}^{\infty} h_{\alpha x} .$$

Replacing the order of summation and integration in this relation and taking into account that the integral of the field $h_{\alpha x}$ of an arbitrary particle α over the period of the structure above the central particle ($\alpha = 0$) is equal to the integral of the field of the central particle over the period of the structure above the particle α, i.e.,

$$\int_{-s/2}^{s/2} h_{\alpha x}\, dx = \int_{s(\alpha-1/2)}^{s(\alpha+1/2)} h_{0x} dx ,$$

we find

$$\bar{\varphi}_2 (z) = -\frac{P}{s}\int_{-\infty}^{\infty} h_{0x} dx .$$

Computation of the integral in this relation yields a zero result, so that the interference component exerts no influence on the average magnetophoretic potential in motion of the suspension along the x axis. In the case of transverse magnetization the contribution of the interference component to the average-along-x potential also turns out to be equal to zero. Thus, if the suspension flows in the x direction, the magnetic structure above the layer surface ($z > 1$) attracts paramagnetic particles and repels diamagnetic particles in equal measure in longitudinal and transverse magnetization. The diagram of the filter with flow and magnetization along the x axis can be used for confinement of paramagnetic particles in the filter. For confinement of diamagnetic particles one can use periodic regimes including alternating cycles of sedimentation at rest and flow in continuous transverse magnetization.

Let us consider in greater detail the properties of the average magnetophoretic force $\bar{f}_m = -d\bar{\varphi}/dz$. Figure 2 shows the dependence of its absolute value (the average force is negative, which means the attraction of paramagnetic particles to the structure) on the distance for different values of the structural step s. (We recall that once the magnetic saturation of the cylinders is attained, the average potential is independent of the intensity of the field applied.) Figure 3 gives the dependence of the average force on the structural step at the distances z = 1, 1.5, and 2. As follows from the data in Fig. 2, the value of the average force decreases exponentially, in practice, with increase in the distance. Sparseness of the structure (increase in s) leads to a broadening of the zone of action of the magnetophoretic force. When the sparseness is small, the absolute value of the average force on the plane $z = 1$ increases but then decreases, passing through the maximum for $s \approx 2.75$. At the distance from the plane of the cylinders' axes, equal to the cylinder diameter, the average force is equal to zero, in practice, in the case of a dense packing and it comprises a considerable value when the structural step increases to $s = 4$ (Fig. 3). We can infer that the optimum step of the structure in separation of paramagnetic particles in the transverse flow is three to four radii of the cylinder. This conclusion is tentative in character, since we have not considered the features of separation due to the hydrodynamics of the suspension in different methods of organization of its flow through the filter.

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NOTATION

S, step of packing of ferrocylinders, cm; a, cylinder radius, cm; s, dimensionless step of packing of ferrocylinders, $s = S/a$; $\mathbf{H}_0$, intensity of the external magnetic field, Oe; $\mathbf{e}$, unit vector in the direction of the external magnetic field; M_s, saturation magnetization, G; P, dimensionless field intensity, $P = H_0/(\pi M_s)$; α, cylinder No.; X , Y, Z, Cartesian coordinates; x, z, dimensionless coordinates (in cylinder radii); $\hat{x}$, $\hat{z}$, dimensionless coordinates measured by the structural step; $\mathbf{i}$ and $\mathbf{k}$, unit vectors of the Cartesian coordinates system on the x and z axis; R, radius vector; $\mathbf{r}_{\alpha A}$, dimensionless radius vector from the cylinder axes to the point A(x, z); χ, magnetic susceptibility of the separated particles; χ_0, magnetic susceptibility of the carrier liquid; v, particle volume, cm^3; $\Delta\chi = \chi - \chi_0$; Φ and φ, dimensional and dimensionless magnetophoretic potentials; φ_1 and φ_2, components of the magnetophoretic potential; $\bar{\varphi}$, average dimensionless magnetophoretic potential; $\mathbf{F}$, force, g·cm·sec^{-1}; $\mathbf{f}$, dimensionless force; A, B, N, T, and I, functions of the dimensionless coordinates $\hat{x}$ and $\hat{z}$. Subscripts: s, saturation; m, magnetization.

REFERENCES

1. C. Latour and H. Kolm, *IEEE Trans. Magnet.*, Vol. Mag-11, 1570–1572 (1975).
2. C. G. Owen, *Biophys. J.*, **22**, 171–178 (1978).
3. É. Ya. Blum, Yu. A. Mikhailov, and R. Ya. Ozols, *Heat and Mass Transfer in a Magnetic Field* [in Russian], Riga (1980).
4. G. Mare and K. Dransfeld, in: F. Gerlach (ed.), *Strong and Superstrong Magnetic Fields and Their Application* [Russian translation], Moscow (1988), pp. 180–262.
5. A. V. Sandulyak, *Magnetofiltration Purification of Liquids and Gases* [in Russian], Moscow (1988).
6. A. J. Richards, T. E. Thomas, O. S. Roath, et al., *J. Magnetism Magnetic Mater.*, **122**, 364–366 (1993).
7. N. G. Moshechkov, R. S. Makhlin, A. Yu. Baryshnikov, et al., in: *Collection of Sci. Papers presented at 10th Jubilee Int. Pless Conf. on Magnetic Fluids* [in Russian], Pless (2002), pp. 343–346.
8. S. N. Podoinitsyn, A. A. Kuznetsov, E. A. Simanovich, et al., in: *Ext. Abstr. of Papers presented at 4th All-Union Conf. on Magnetic Fluids* [in Russian], Ivanovo (1985), p. 43.
9. B. É. Kashevskii and I. V. Prokhorov, *Inzh.-Fiz. Zh.*, **76**, No. 4, 30–35 (2003).